Fiber orientation mapping in an anisotropic medium with NMR diffusion spectroscopy

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Purpose: The diagonal and off-diagonal components of the apparent self-diffusion tensor, \( D \), are used to construct a diffusion ellipsoid for a voxel that depicts both orientation of tissue microstructures, such as muscle fibers, and mean diffusion distances. New weighting parameters are also suggested for structural NMR imaging.

Principles: In heterogeneous, anisotropic media, \( D \) relates the flux of spin-labeled protons to their concentration gradient. Since \( D \) is symmetric and positive-definite, its three mutually orthogonal eigenvectors, \( e_1, e_2, \) and \( e_3 \), define the principal axes and its three positive eigenvalues, \( \lambda_1, \lambda_2, \) and \( \lambda_3 \), are the diffusivities in these directions [2]. Continuum models of diffusion in heterogeneous media [3] suggest that the principal axes of \( D \) coincide with those of the grain or fiber and the principal diffusivities of \( D \) are related to the structure, geometry, and diffusivity of the various microscopic compartments within the heterogeneous medium.

The diffusion ellipsoid: In heterogeneous, anisotropic media [3], the macroscopic effective self-diffusion tensor, \( D \), appears in the conditional probability density function, which is the probability that a particle at \( x \) at time \( t \) was at \( x_0 \) at \( t = 0 \):

\[
\frac{x(x_0,t)}{\sqrt{\mid D \mid (4\pi t^3)}} = \frac{1}{\sqrt{\mid D \mid (4\pi t^3)}} \exp\left(\frac{-(x-x_0)^T D^{-1} (x-x_0)}{4t}\right),
\]

setting the quadratic form to 1/2, i.e.,

\[
\frac{(x-x_0)^T D^{-1} (x-x_0)}{2t} = 1,
\]

defines a diffusion ellipsoid whose principal axes constitute the local "fiber" frame of reference, and whose \( j^{th} \) major axis is the mean distance a spin-labeled proton diffuses in the \( j^{th} \) principal direction, \( \sqrt{\lambda_j t} \), during the diffusion time, \( t \).

The scalar invariants: Three scalar invariants of \( D \), \( \lambda_1, \lambda_2, \) and \( \lambda_3 \), are:

\[
\begin{align*}
1 &= \lambda_1 + \lambda_2 + \lambda_3 = \text{Tr} D \\
2 &= \lambda_1 \lambda_2 + \lambda_3 \lambda_1 + \lambda_2 \lambda_3 \\
3 &= \lambda_1 \lambda_2 \lambda_3 = |D|.
\end{align*}
\]

They have the desirable properties of being independent of the coordinate system in which \( D \) is measured, and insensitive to the scheme by which \( \lambda_j \) are numbered, making them (or functions of them), ideal weighting factors in structural NMR imaging.

Data analysis: Diffusion ellipsoids for pork loin are constructed from two apparent self-diffusion tensors, \( D^{01} \) and \( D^{41} \), estimated from spin-echo experiments [1]:

In Fig 1a, the grain of the sample was nearly aligned with the magnet's x axis. Eigenvalues (principal diffusivities) of \( D^{01} \) are \( \lambda_1 = (1.0406 \pm 0.0007) \times 10^{-5}, \lambda_2 = (0.944 \pm 0.001) \times 10^{-5}, \lambda_3 = (0.8532 \pm 0.0006) \times 10^{-5} \) (cm²/sec).

In Fig. 1b, the same sample is rotated approximately 41° in the z-x plane. Eigenvalues of \( D^{41} \) are \( \lambda_1 = (1.0119 \pm 0.0003) \times 10^{-5}, \lambda_2 = (0.9343 \pm 0.0006) \times 10^{-5}, \lambda_3 = (0.8767 \pm 0.0018) \times 10^{-5} \) (cm²/sec).

Discussion: The eigenvectors that define the fiber frame follow the sample when it is rotated. This is represented by the tipping of the polar axis. The scalar invariants of \( D \) differ by no more than 1% in both cases because they are intrinsic to \( D \), independent of the sample's orientation in the magnet. Both ellipsoids are nearly spherical, presumably because the diffusion time, \( \Delta = 22.5 \) ms, corresponding to a mean diffusion distance of 4.7 µm, is too short for the majority of spin-labeled protons to encounter diffusional barriers.

Although a single voxel was used in this study, these principles can be generalized to multiple voxels. One could envision 3-D fiber maps [4] or diffusion ellipsoids displayed in each voxel, connected like link sausages that follow fiber tracts.

Conclusion: Constructing the diffusion ellipsoid requires knowledge of all diagonal and off-diagonal elements of \( D \). Inherently, \( D \) contains unique directional, structural and anatomical information within a voxel that scalars such as \( T_1 \) or \( T_2 \) do not - information that is embodied in the diffusion ellipsoid and scalar invariants.

References: