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Quantifying Errors in Fiber-Tract Direction and Diffusion Tensor Field Maps Resulting from MR Noise

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INTRODUCTION

Diffusion tensor MRI (DT-MRI) provides a discrete representation of the diffusion tensor field of water in living tissues. A graphical means to display the diffusion tensor field, $\underline{D}(x,y,z)$, is to construct diffusion ellipsoid maps or images (1). Moreover, from the diffusion tensor field, one can infer the fiber-tract direction field in some anisotropic tissues (1). In white matter and skeletal muscle, for example, the local fiber-tract direction corresponds to the major or polar axis of the ellipsoid, and is thus given by the principal direction (eigenvector) $\varepsilon_1(x,y,z)$, associated with the largest principal diffusivity (eigenvalue), $\lambda_1(x,y,z)$ (1). This vector field can be displayed in various ways, e.g., as an arrow or a line drawn in each voxel (2) or as a set of Euler angles displayed in each voxel (3).

One issue of great biological significance, but not previously addressed, is the degree of susceptibility of diffusion tensor fields, and their associated fiber-tract direction maps, to MRI noise present in diffusion weighted (amplitude) images (DWIs). Since DT-MRI is intrinsically a statistical technique (4), $\underline{D}$ and all of the quantities computed from it are random variables. Therefore, the eigenvalues and eigenvectors in each voxel are not determined precisely in a DT-MRI experiment, but are subject to variation. Moreover, MRI noise has already been shown to introduce a significant bias in the assignment of the eigenvalues of $\underline{D}$ (5). Since each eigenvector is associated with a particular eigenvalue, the assignment or determination of the fiber-tract direction in each voxel (or of the set of Euler angles) is also biased.

An added complication in addressing this question is the inherently non-linear, non-analytical relationship between $\underline{D}$ and quantities computed from it, and the set of DWIs from which $\underline{D}$ is estimated (4). Therefore, we must resort to empirical approaches, such as Monte Carlo simulations, and approximate methods, such as Matrix Perturbation Analysis to assess these complex relationships.

METHODS

Monte Carlo simulations are useful in simulating the outcome of DT-MRI experiments subject to different levels of background thermal noise (5). Typically, we synthesize noisy DWIs (5) by assuming a distribution of diffusion tensors that are representative of ones we measure in different regions of living tissue, and assuming values of b-matrices and MRI parameters that are identical to those used in those studies, as well as an assumed level of background (thermal) Gaussian noise (6). From the noisy magnitude DWIs, we then estimate $\underline{D}$ in each voxel as in (4), from which we calculate its eigenvalues and eigenvectors, as well as their statistical distributions.

First-Order Matrix Perturbation Analysis (7) provides another means to estimate the mean and variance of the eigenvalues and eigenvectors of $\underline{D}$, directly from the estimated $\underline{D}$ and its estimated covariance matrix, $\Delta\underline{D}$ that contains the standard errors of each component of $\underline{D}$ (4). The uncertainty in each eigenvalue λ_j is $\Delta\lambda_j$, and the uncertainty in the matrix of eigenvectors, $\underline{E}$, is $\Delta\underline{E}$, where

$$\Delta\lambda_j \cong \varepsilon_j^T \Delta\underline{D}\, \varepsilon_j ;\ \Delta\underline{E} \cong \underline{E}\,\underline{R} \text{ and } R_{ij} = \frac{\varepsilon_i^T \Delta D \varepsilon_j}{\lambda_i - \lambda_j}\left(1-\delta_{ij}\right)$$

ε_i is the i^{th} eigenvector, and δ_{ij} is the Kronecker delta. The subtended angle, $\Delta\theta_i$, between the i^{th} perturbed eigenvector of $\underline{D}$, $\varepsilon_i+\Delta\varepsilon_i$, and the estimated eigenvector ε_i, measures the angular deviation of the fiber direction, $\Delta\theta_i$:

$$\Delta\theta_i = \tan^{-1}(\,|\Delta\varepsilon_i|)$$

RESULTS AND DISCUSSION

Figure 1 shows a graphical construct that should be useful in displaying both the eigenvectors of $\underline{D}$ and their associated uncertainties. Matrix Pertubation Analysis shows that to first order, the unit eigenvector, ε_i, is orthogonal to its uncertainty vector, $\Delta\varepsilon_i$. This suggests that we can display the unit eigenvector and with "cone of uncertainty" around its tip. This would convey the fact that the fiber direction is not known precisely, which is currently implied fiber orientation images.

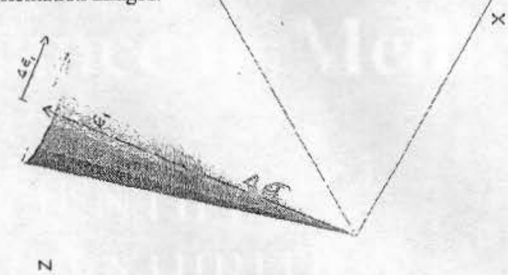

Fig 1: Eigenvector and its cone of uncertainty

CONCLUSIONS

We show that background noise in the DWIs affects the determination of the fiber tract direction calculated from the diffusion tensor. Monte Carlo simulations of Diffusion Tensor Imaging experiments explain the origin of the statistical bias and Matrix Pertubation methods provide good order of magnitude estimates of the variance in the eigenvectors. More sophisticated algorithms must be developed to eliminate the sorting bias in the eigenvalues and eigenvectors of $\underline{D}$, perhaps by using information about local fiber direction field coherence or order (8) or by using invariants of the diffusion tensor to perform the sorting (9).

It is essential that studies intended to determine fiber directional patterns, such as those presently being conducted to elucidate muscle fiber architecture in cardiac tissue (2) and nerve fiber direction fields in brain (3), will begin to address and account for MR noise-induced artifacts in fiber-direction, and that careful histological analysis accompany studies of fiber direction fields to ensure that no systematic artifacts corrupt the analysis of the diffusion tensor data.

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