Visualization and Processing of Tensor Fields

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Preface

Matrix-valued data sets – so-called tensor fields – have gained significant importance in the fields of scientific visualization and image processing. The tensor concept is a common physical description of anisotropic behaviour, especially in solid mechanics and civil engineering. It arises e.g. in the measurements of stress and strain, inertia, permeability and diffusion. In the field of medical imaging, diffusion tensor magnetic resonance imaging (DT-MRI) has become widespread in order to gain valuable insights into connectivity properties of the brain. Tensors have also shown their use as feature descriptors in image analysis, segmentation and grouping.

These recent developments have created the need for appropriate tools for visualizing and processing tensor fields. Due to the multivariate structure of the data and their multidimensional variation in space, tensor field visualization belongs to the most challenging topics in the area of scientific visualization. Moreover, most signal and image processing methods have been developed for scalar- and vector-valued data sets, and only recently researchers have tried to investigate how they can be extended to tensor fields. In this case one has to take into account a number of additional constraints such as an appropriate coupling of the different channels, and preservation of properties such as positive semidefiniteness during the filtering process.

Unfortunately the results in the field of tensor-valued visualization and image processing are scattered in the literature, and often researchers in one application area are not aware of recent progress in another area. In order to address this problem, the editors of this book organized a perspective workshop that took place at Schloss Dagstuhl, Germany from April 18 to 23, 2004. In that week 30 invited scientists, representing many of the world-wide leading experts in tensor field visualization and processing, had the opportunity to exchange ideas in a highly inspiring atmosphere. Since many of them met for the first time, this exchange proved to be particularly fruitful.

One of these fruits was the wish of all participants to compile their knowledge in a single edited volume. The present book – which is the first of its kind in this field – is the result of these efforts. Its goal is to present the state-of-the-art in the visualization and processing of tensor fields, both as an overview for the inquiring scientist, and as a basic foundation for developers and practitioners. The book contains some longer chapters dedicated to surveys and tutorials of specific topics, as well as a great deal of original work not previously published. In all cases the emphasis has been on presenting the details necessary for others to reproduce the techniques and algorithms. Another goal of this book is to provide the basic material for teaching state-of-the-art techniques in tensor field visualization and processing. It can therefore also serve as a textbook for specialized classes and seminars for graduate and doctoral students. An extended bibliography is included at the end of each chapter pointing out where to obtain further information.

Organization of the Book

This volume consists of 25 chapters. Each of them has been reviewed by two experts and carefully revised according to their suggestions. The chapters are arranged in five thematic areas. Color plates can be found in the Appendix.

The book starts with an introductory chapter by Hagen and Garth. It gives the mathematical background from linear algebra and differential geometry that is necessary for understanding the concept of tensor fields.

Part I of the book is devoted to feature detection using tensors. Here one typically starts with scalar- or vector-valued images and creates tensor-valued features that are suitable for corner detection, texture analysis or optic flow estimation. Structure tensors are the most prominent representatives of these concepts. Chapter 2 by Brox et al. is a survey chapter on adaptive structure tensors, while Chap. 3 by Nagel analyzes closed form solutions for a structure tensor concept in image sequence analysis. An alternative framework for tensor-valued feature detection is presented in Chap. 4 by Köthe who shows that the so-called boundary tensor may overcome some problems of more traditional approaches.

Part II deals with the currently most important technique for creating tensor-valued images, namely Diffusion Tensor Magnetic Resonance Imaging (DT-MRI), often simply called Diffusion Tensor Imaging (DTI). This technique measures the diffusion properties of water molecules and has gained significant popularity in medical imaging of the brain. Chapter 5 by Alexander gives a general overview of the principles of biomedical diffusion MRI and algorithms for reconstructing the diffusion tensor fields, while the subsequent chapter by Hahn et al. describes the empirical origin of noise and analyzes its influence on the DT-MRI variables. After having understood the formation of DT-MR images their adequate visualization remains a challenging task. This is the topic of the survey chapter by Vilanova et al. that also sketches the clinical impact of DT-MR imaging. A more specific medical application is treated in Chap. 8 by Gee et al. who study anatomical labeling of cerebral white matter in DT-MR images. For conventional DT-MR imaging, the identification and analysis of fibre crossings constitutes a severe difficulty. In Chap. 9, Pasternak et al. introduce a variational image processing framework for resolving these ambiguities. An alternative strategy is investigated in Chap. 10 by Özarslan et al. They reconstruct higher-order tensors from the MR measurements. These allow to encode a richer orientational heterogeneity.

In the third part, general visualization strategies for tensor fields are explored. This part starts with a review chapter by Benger and Hege who also consider applications in relativity theory. Kindlmann's chapter is concerned with visualizing discontinuities in tensor fields by computing the gradients of invariants. The subsequent Chaps. 13–16 investigate different strategies for understanding the complex nature of tensor fields by extracting suitable differential geometric information. While Chap. 13 by Tricoche et al. deals with the topology and simplification of static and time-variant 2-D tensor fields, Chap. 14 by Zheng et al. is concerned with a novel and numerically stable analysis of degerated tensors in 3-D fields. In Chap. 15, Wischgoll and Meyer investigate the detection of alternative topological features, namely closed hyperstreamlines. The third part is concluded with a chapter by Hotz et al. who introduce specific visualization concepts for stress and strain tensor fields by interpreting them as distortions of a flat metric.

Part IV of the book is concerned with transformations of tensor fields, in particular interpolation and registration strategies. In Chap. 17, Moakher and Batchelor perform a differential geometric analysis of the space of positive definite tensors in order to derive appropriate interpolation methods. The next chapter by Pajevic et al. deals with non-uniform rational B-splines (NURBS) as a flexible interpolation tool, while in Chap. 19 Weickert and Welk introduce a rotationally invariant framework for tensor field approximation, interpolation and inpainting. It is based on partial differential equations (PDEs). Finally, Chap. 20 by Gee and Alexander treats the problem of diffusion tensor registration. Compared to scalar-valued registration approaches, incorporating the orientation information provides additional challenges.

The fifth part is a collection of contributions on signal and image processing methods that are specifically developed to deal with tensor fields. Chapter 21 by Welk et al. as well as Chap. 22 by Burgeth et al. show that seemingly simple ideas like median filtering and morphological image processing can create substantial difficulties when one wants to generalize them to tensor fields. Since tensor lack a full ordering, many straightforward concepts cannot be applied and alternative generalizations become necessary. In Chap. 23, Suárez-Santana et al. review adaptive local filters for tensor field regularization and interpolation that are steered by a structure tensor, while Chap. 24 by Westin et al. is concerned with two other regularization techniques for tensor fields: normalized convolution and Markov random fields. These ideas a complemented by Chap. 25 where Weickert et al. survey the most important PDE approaches for discontinuity-preserving smoothing and segmentation of tensor fields.

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Saarbrücken and Kaiserslautern, *Joachim Weickert*
June 2005 *Hans Hagen*

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